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In: Jan Chleboun and Petr Přikryl and Karel Segeth (eds.): Programs and Algorithms of Numerical Mathematics, Proceedings of Seminar. Dolní Maxov, June 6-11, 2004. Institute of Mathematics AS CR, Prague, 2004. pp. 76–81.

Persistent URL: http://dml.cz/dmlcz/702778

NUMERICAL SOLUTION OF INVISCID INCOMPRESSIBLE FLOW IN A CHANNEL WITH DYNAMICAL EFFECTS *

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Abstract

Numerical solution of unsteady 2D inviscid incompressible flows described by Euler equations over the vibrating profile NACA 0012 in a channel is studied. The finite volume method (FVM) and a higher order cell-centered scheme with an artificial dissipation at a qudrilateral C-mesh is used. The method of artificial compressibility and the time dependent method are used for steady state solutions. Numerical results are compared with experimental data.

1. Introduction

The work resumes the article presented in [3]. The Lax-Wendroff scheme in Richtmyer form of second order accuracy with an added artificial viscosity is used for the numerical solution of flow over the profile NACA 0012 in a channel.

2. Mathematical model

The governing system of equations is the system of Euler equations for 2D inviscid incompressible flow in conservation form:

$$\tilde{R}W_t + F_x + G_y = 0 \ , \tag{1}$$

where $W = \left(\frac{p}{\rho}, u, v\right)^T$, $F = \left(u, u^2 + \frac{p}{\rho}, uv\right)^T$, $G = \left(v, uv, v^2 + \frac{p}{\rho}\right)^T$, $\tilde{R} = \operatorname{diag} \|0, 1, 1\|$, ρ is the density (constant), p is the pressure and (u, v) is the velocity vector.

The system (1) is solved with the diagonal matrix $\tilde{R} = \operatorname{diag} \left\| \frac{1}{a^2}, 1, 1 \right\|$, $a \in R$, which represents the method of artificial compressibility used for steady state solutions.

Upstream conditions are $u = u_\infty$, $v = v_\infty$, p is extrapolated. Downstream condition is only given by $p = p_2$. Next values of the vector of conservative variables are extrapolated at the outlet. Wall condition on fixed walls of the channel is the impermeability condition, i. e., $(u, v)_n = 0$ (the normal component of the velocity vector is equal to zero). Two approaches are applied to wall conditions on oscillating profile sides in the channel. At first, the impermeability condition is applied. At second, the velocity vector of the flow field close to profile sides is determined by the

*This work was supported by grants No. 201/02/0684 of the Grant Agency of the Czech Republic, Research Plan MSM 210000010 and 101/02/0391 of the Grant Agency of the Czech Republic.

velocity vector normal to the profile boundary and given by the angular velocity of the profile.

The method of artificial compressibility and the time dependent method are used for computations of steady state solutions. In the case of unsteady solution it is necessary to consider $a \to \infty$ or $a \gg K$, where K is a given (large) positive number.

2.1. Prescribed profile oscillation

The motion of the profile fixed in the point of an elastic axis is given by the formula

$$\varphi = \varphi_0 \sin(2\pi f t) \ , \tag{2}$$

where φ $[rad]$ is the angle of rotation of the profile from the equilibrium position, φ_0 is the amplitude of oscillations, f $[s^{-1}]$ is the frequency and t $[s]$ is time.

3. Numerical solution

3.1. Numerical scheme in FVM

The cell-centered Lax-Wendroff (Richtmyer form) scheme in a form of predictor-corrector with an added artificial viscosity is used at quadrilateral C-mesh ($m = 4$):

- LW Richtmyer predictor:

$$W_i^{n+1/2} = W_i^n - \frac{1}{2}\frac{\Delta t}{\mu_i}\sum_{k=1}^{m}(\bar{F}_{ik}^n \Delta y_k - \bar{G}_{ik}^n \Delta x_k) + \frac{\varepsilon}{m}\sum_{k=1}^{m}(W_k^n - W_i^n)$$

- LW Richtmyer corrector:

$$\begin{aligned}
\tilde{W}_i^{n+1} &= W_i^n - \frac{\Delta t}{\mu_i}\sum_{k=1}^{m}(\bar{F}_{ik}^{n+1/2}\Delta y_k - \bar{G}_{ik}^{n+1/2}\Delta x_k) \ , \\
W_i^{n+1} &= \tilde{W}_i^{n+1} + \mathrm{AD}W_i^n,
\end{aligned}$$

  where $\mathrm{AD}W_i^n$ is artificial viscosity

  and $\bar{F}_{ik} = \frac{1}{2}(F_i + F_k), \bar{G}_{ik} = \frac{1}{2}(G_i + G_k), \varepsilon \in (0, 1\rangle$.

3.2. Wall conditions

Wall conditions are realized by using the “reflection principle”.

Wall conditions on the oscillating profile are realized by using “small disturbance theory” because only small changes of the angle of attack φ are considered ($|\varphi| \le 6^0$). Another method how to treat the oscillating profile is to employ the Arbitrary Lagrangian-Eulerian method, see, e. g., [6].

Two different approaches to the numerical realization of the wall conditions on the oscillating profile are applied: the impermeability condition is used in every time, or the following condition is used:

$$\left(-\frac{df}{dx}\right) u + v = \dot{\varphi} f(x) \left(-\frac{df}{dx}\right) + \dot{\varphi}(e - x) \ , \tag{3}$$

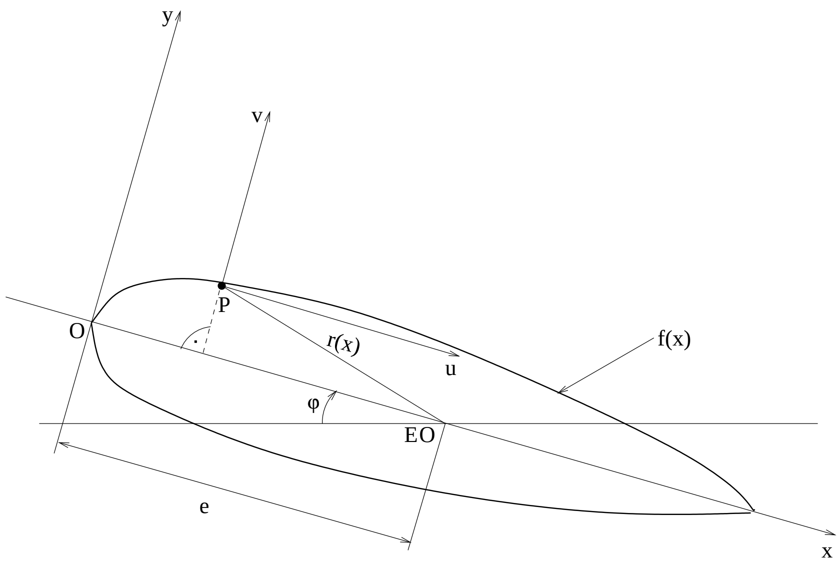

Fig. 1: *Meaning of symbols in the equation (3).*

where (u, v) is the velocity vector, $\dot{\varphi}$ is the angular velocity for the profile rotation, $f(x)$ is a function describing the shape of the profile surface and e is the distance of the elastic axis EO from the origin O of a local coordinate system.

All quantities in the equation (3) are related to the local coordinate system with the origin O (see Figure 1).

Dimensionless variables are used for the numerical solution. Relations between dimensional and dimensionless variables are given by the normalization of dimensional variables with reference (dimensional) values, see, e. g., [2].

4. Numerical results

Results of numerical solution of flow over the vibrating profile NACA 0012 in the channel are presented. Steady state results and their comparison with experimental data are presented, e. g., in [2].

Figures 2, 3 and 4 present results for the prescribed oscillation of the profile NACA 0012 given by the formula (2). The elastic axis is situated in 25% of the profile chord from the local coordinate system origin. The legend **phi/10** in Figure 2 and 3 denotes a distribution of the variable φ according to the equation (2).

Figure 2 shows the pressure coefficient C_p during three oscillation periods at points $\frac{x}{c} = 0.2$ on the upper ($C_{p_{\text{up}}}$) and lower ($C_{p_{\text{down}}}$) side of the oscillating profile, where c is the length of the profile chord. The oscillation frequency is $f = 30$ Hz and the amplitude $\varphi_0 = 3^o$. Results are compared for two different values of the parameter a, namely $a = 1$ and $a = 10$. The wall conditions are the impermeability conditions. These conditions don't reflect the influence of the elastic axis location of the profile.

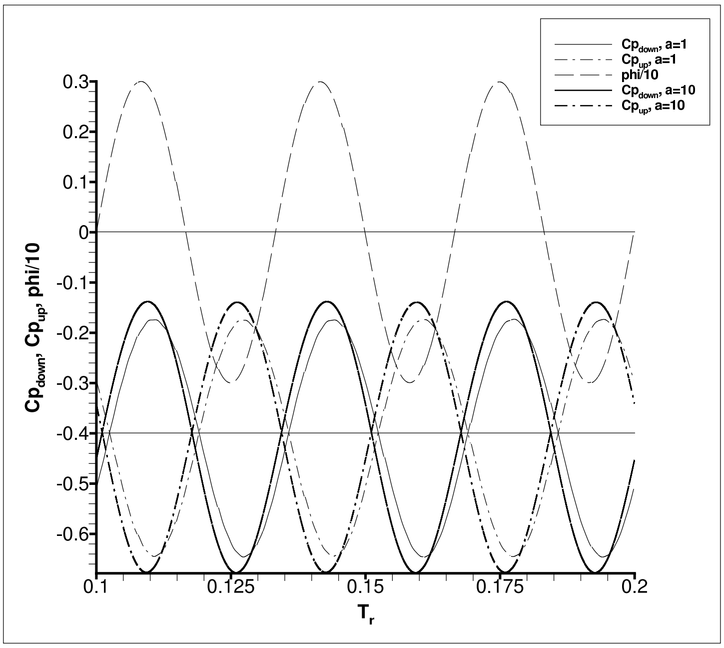

Fig. 2: *C_p in points $\frac{x}{c} = 0.2$ for the oscillating profile with $f = 30Hz$, comparison of computed results for $a = 1$ and $a = 10$.*

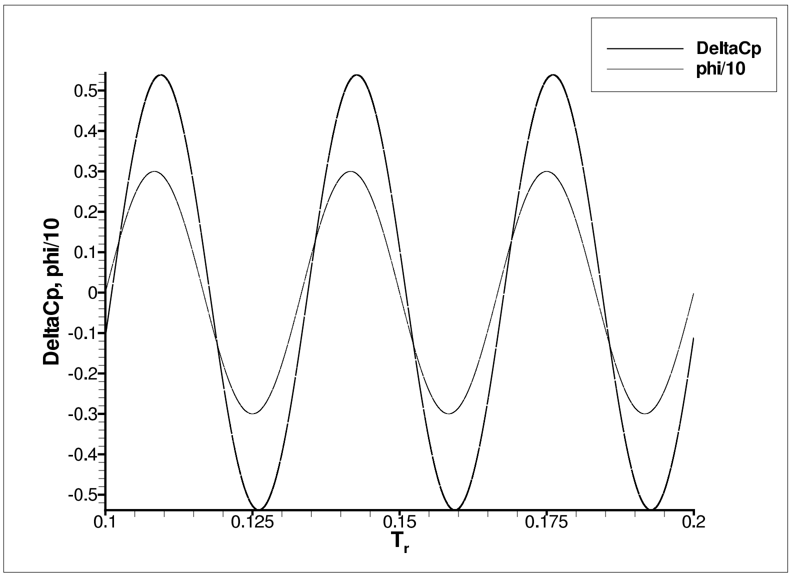

Fig. 3: *ΔC_p for the oscillating profile with $f = 30Hz$, $a = 10$.*

Figure 3 shows $\Delta C_p = C_{p_{\rm down}} - C_{p_{\rm up}}$ for points with $\frac{x}{c} = 0.2$. These results are suitable for comparison with available experimental data [1].

Table 1 presents the comparison of computed, experimental and theoretical data for oscillating components (real and imaginary part) of ΔC_p for points on the profile

	Re ΔC_p	Im ΔC_p
experimental data (Triebstein)	0.494	-0.0325
theoretical data (Triebstein)	0.440	-0.114
computed values	0.529	-0.108

Tab. 1: *Comparison of computed, experimental and theoretical data for Re ΔC_p and Im ΔC_p.*

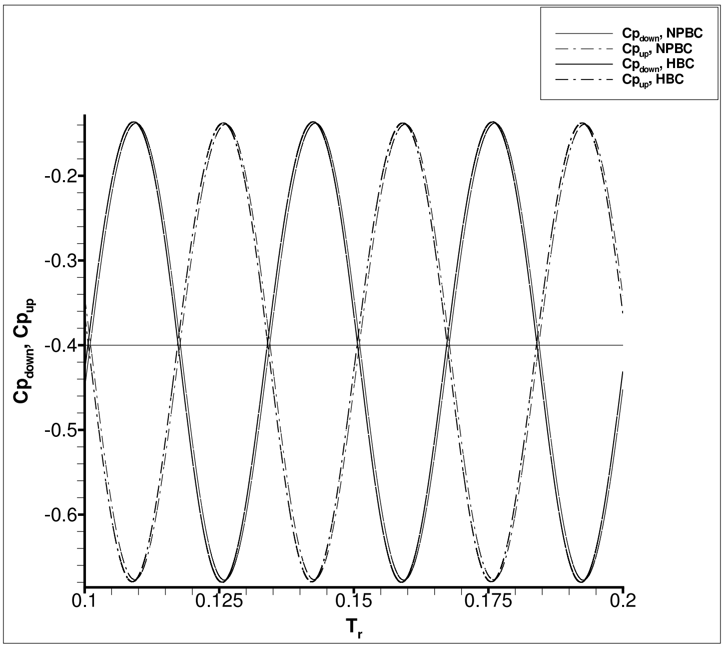

Fig. 4: *C_p in points $\frac{x}{c} = 0.2$ for the oscillating profile with $f = 30Hz$, comparison of computed results for both applied boundary conditions.*

surface with $\frac{x}{c} = 0.2$. The oscillation frequency of the profile is $f = 30$Hz and the amplitude is $\varphi_0 = 3^o$. Experimental and theoretical data are data of Triebstein [7] converted with respect to the amplitude of 3 degrees.

Figure 4 compares C_p plotted for points on the oscillating profile surface with $\frac{x}{c} = 0.2$ for both boundary conditions applied. Boundary conditions (3) involve the influence of the elastic axis position instead of the simple impermeability boundary condition. Only small differences are observed in results obtained for these two boundary conditions applied.

5. Conclusion

A numerical solver of steady and unsteady inviscid incompressible flow over a profile in a channel was developed. An explicit scheme with an added artificial viscosity in FVM was used. The attention was particularly paid to the influence of the parameter a value and to the comparison of different boundary conditions applied to the surface of the oscillating profile.

Table 1 shows that the data computed for the oscillating profile are comparable with Triebstein's experimental and theoretical data [7].

The presented results indicate that the developed numerical solver can be used for engineering numerical simulations.

However, in the case of the unsteady solution, the influence of the parameter a value (see matrix $\tilde{R}$ in (1)) on the solution becomes a problem. This can be seen in Figure 2. Therefore, it is intended to avoid this influence by the introduction of another method treating the unsteady solution. The application of a dual time stepping method or an implicit scheme is the intended approach.

It is intended to compare the computed data with results obtained through different numerical methods. One of them is the finite element method (FEM) applied to the same problem. For details of an application of FEM on problems of aeroelasticity see, e. g., [4].

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